Problems: Section 0.1

1. Display all subsets of
   a) the set $S = \{x, y, z\}$;
   b) the set $\{2, 4, 6, 8\}$.
2. Display the powerset of the powerset of $\{a\}$.
3. Describe in English each of the following sets.
   a) $\{m \mid m = 2n \text{ for some } n \in Z\}$.
   b) $\{s \mid s = n^2 \text{ for some } n \in N^+\}$.
   c) $\{q \mid q = mn \text{ for some } m, n \in N,\ m, n > 1\}$.
4. Use set builder notation to describe the set of
   a) all odd numbers between 100 and 200;
   b) all points on the graph of the function $y = x^2$.
5. Let N, N^+, Z, and Q be as defined in the text. Let E be the set of even integers and F the set of integers divisible by 5. (Note that $0, -5 \in F$.) Use set operators to name the set of
   a) positive integers divisible by 5;
   b) even integers divisible by 5;
   c) nonintegral rational numbers;
   d) integers divisible by 10;
   e) positive integers divisible by 10;
   f) odd positive integers;
   g) odd negative integers;
   h) nonpositive integers;
   i) negative integers;
   j) ordered pairs, where the first entry is even and the second odd.
6. Let $R = \{1, 2, 3, 4\}$, $S = \{1, 3, 5, 7, 9\}$, and $T = \{3, 4, 5, 6, 7, 8\}$. Compute:
   a) $R \cup S$, $R \cup T$, and $S \cup T$.
   b) $R \cap S$, $R \cap T$, and $S \cap T$.
   c) $R - S$, $R - T$, and $S - T$.
   d) the complements of R, S, and T assuming the universe is the set of all integers from 1 to 10.
   e) $R \cap S \cup T$.
   f) $R - S \cap T$.
   g) $R - S \cap T - S$.
   h) $R \cap S \cap T$.
7. (a)-(h) Draw Venn diagrams to illustrate each of the expressions in [6].
8. List all the elements of $\{b, c, d\} \times \{e, o\}$.
9. Is the Cartesian product commutative? That is, is $S \times T$ the same as $T \times S$? Why?
10. Let R, S, and T be sets and let U denote the universe. Prove the following equalities using for each either a logical argument in English or a Venn diagram.
    a) $U - R =$ the complement of R.
    b) $R - (S \cup T) = (R - S) \cap (R - T)$.
    c) $R - (S \cap T) = (R - S) \cup (R - T)$.
    d) $R \cap (S - R) = \emptyset$.
11. If $A \cap B = \emptyset$, what does that tell you about $|A \cup B|$?
12. If $A \subset B$, what does that tell you about $A \cap B$ and $A \cup B$?
13. If $A \cap B = \emptyset$ and $B \cap C = \emptyset$, what does that tell you about $A \cap C$?
14. If $A \cup B = D$, must $D - B = A$? If not, what can you conclude if $A \cup B = D$ and $D - B = A$?
15. If $A \subset S$ and $B \subset S$, what can you say about $A \cup B$?
16. If $|A| = m$, $|B| = n$, and $m \geq n$, what is the least $|A \cup B|$ can be? The most?
17. Consider $|A \cap B|$ and answer the questions posed in [16].
18. Find the cardinality of the set
    $$S = \{p/q \mid p, q \in N^+,\ p, q \leq 10\}.$$
19. Starting from the definition that $A \cup B$ is the set of all things which are in at least one of A and B, give an argument in English why $(X \cup Y) \cup Z$ means the set of all things which are in at least one of X, Y, and Z. (*Hint*: Begin by using the definition to get a complicated English sentence describing $(X \cup Y) \cup Z$ and then use your understanding of English to simplify the sentence.)
20. a) Using the same method as in [19], show that $X \cup (Y \cup Z)$ is also the set of all things in at least one of X, Y, and Z. Thus set union is associative. (By induction—see Chapter 2—we could go on and prove that parentheses make no difference when unioning n sets: Any order gives the set of things which are in at least one of the n sets.)
    b) Similarly, show that intersection is associative.